Nassau County Interscholastic Mathematics League

Solutions **2002-2003**
Contest # 3

13. (2x)(x)-(x)(1) = 3. So we get a quadratic $2x^2 - x - 3 = 0$. This factors into $(2x+3)(x-1) = 0$ So $x = -1$ or $\frac{3}{2}$

14. Use the angle bisector theorem on ΔDBC to get $\frac{8}{DE} = \frac{6}{3}$ so DE = 4 Now use the theorem and DC = 7. on ΔADC to get $\frac{AD}{4} = \frac{7}{6}$ so $AD = \frac{14}{3}$

15. Let the leg be x and the hypotenuse x + 2. So $x^2 + 20^2 = (x+2)^2$. Solve to get x = 99 and the hypotenuse is 101.

16. If they were all distinct, there would be 5! (5 "factorial"), but for the two pairs of identical letters we have to divide by 2! twice, so $\frac{5!}{(2!)(2!)} = 30$ [this idea is likely to show up again]

17. Since $(a+b)(a-b) = 37$, and a, b are both positive integers, and 37 is prime, we must have $\begin{matrix} a+b=37 \\ a-b=1 \end{matrix}$ so a = 19 and b = 18.

18. The number we seek must be one less than a common multiple of 2, 4, 6, and 8 (hence of the LCM = 24), but 2 less than a multiple of 11. Trying numbers one less than multiples of 24, i.e. 23, 47, 71, 95, 119, …., we find that 119 is the first to be 2 less than a multiple of 11.